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Studies on Melt Spinning. III. Velocity Field Within the Thread

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Synopsis

The velocity field within a molten spinning thread was analyzed quantitatively by solving the equations of continuity and momentum for Newtonian liquids. In solving the equations, the viscosity was assumed known and was given by the expression

$$\mu = \mu_0 e^{\beta x}(1 + cr^2)$$

where x and r are distances in cylindrical coordinates. A series solution in velocity v having the expression

$$v = v_0 e^{\alpha x}(1 + a_2 r^2 + a_4 r^4 + a_6 r^6 + \cdots)$$

was obtained when several simplifying assumptions were made on the equations. The series solution was found to converge when $cr^2 < 1$ is satisfied. $\mu_0 e^{\beta x}$ and $v_0 e^{\alpha x}$ above are tangents on semilog paper at $x = x$ to the macroscopic viscosity and velocity profiles $\mu(x)$ and $v(x)$ which were computed separately by means of a technique developed previously by the author.[1,2] The value c was derived from the temperature profile across the thread at $x = x$ computed separately using another technique developed by the author.[3] The above series solution showed numerically that under most conceivable spinning conditions the velocity field within the thread is for practical purposes flat across the thread and, in addition, purely extensional.

INTRODUCTION

To this date, the velocity field within a melt spinning thread has intuitively been assumed flat across the thread and purely extensional by most researchers, except that Ziabicki[4] discussed this problem to some extent qualitatively and Matovich et al.[5] contributed theoretical reasoning by expanding the velocity and other variables into series and substituting them into the equations of continuity and momentum (hereafter called the Navier-Stokes equations for brevity). However, a complete solution of the Navier-Stokes equations giving the velocity field within a melt spinning thread quantitatively has not been available.

In the present study, the author obtained a solution to the Navier-Stokes equations having the expression

$$v = v_0 e^{\alpha x}\ (1 + a_2 r^2 + a_4 r^4 + a_6 r^6 + \ldots)$$

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where v is the velocity in the x-direction and x and r are distances in cylindrical coordinates. The shear viscosity distribution within the thread was assumed known and was given by the formula

$$\mu = \mu_0 e^{\beta x}(1 + cr^2).$$

The above solution gives the steady-state velocity field within the thread at any point along the thread except in the die swell region. Values v_0, α, μ_0, and β are functions of x and can be derived from the macroscopic thread thickness $A(x)$ and thread temperature $t(x)$ which can be computed separately by means of a theoretical technique developed by the author previously.[1,2] The c-value was derived from the temperature profile $t(x, r, \theta)$ across the thread computed by using another technique developed previously by the author.[3]

By determining the values of coefficients a_2, a_4 ... numerically, the validity of approximating the velocity field by a flat velocity profile across the thread and further by a pure extentional flow is discussed.

SOLUTION OF THE NAVIER-STOKES EQUATIONS

The most general form of the Navier-Stokes equations in cylindrical coordinates (x, r, θ) is given, for instance, in McKelvey's *Polymer Processing*.[6] When the following six conditions are met, the Navier-Stokes equations take a simplified form, eqs. (1) through (7): (i) steady state; (ii) constant density; (iii) axial symmetry both in temperature and velocity fields; (iv) negligibility of gravity force; (v) negligibility of inertia forces; (vi) a known temperature profile:

$$\frac{1}{r}\frac{\partial}{\partial r}(rv_r) + \frac{\partial v}{\partial x} = 0 \tag{1}$$

$$\frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rx}) + \frac{\partial \tau_{xx}}{\partial x} = \frac{\partial P}{\partial x} \tag{2}$$

$$\frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rr}) - \frac{\tau_{\theta\theta}}{r} + \frac{\partial \tau_{rx}}{\partial x} = \frac{\partial P}{\partial r} \tag{3}$$

$$\tau_{rr} = 2\mu\frac{\partial v_r}{\partial r} \tag{4}$$

$$\tau_{\theta\theta} = 2\mu\frac{v_r}{r} \tag{5}$$

$$\tau_{xx} = 2\mu\frac{\partial v}{\partial x} \tag{6}$$

$$\tau_{xr} = \mu\left(\frac{\partial v}{\partial r} + \frac{\partial v_r}{\partial x}\right). \tag{7}$$

Stress τ_{rr}, etc., are the differences between the true stresses σ_{rr}, etc., and static pressure P. It is understood that τ is positive when the stress is tensile and P is positive when the pressure is compressive; μ is the shear viscosity:

$$\tau_{rr} = \sigma_{rr} + P \tag{8}$$

$$\tau_{\theta\theta} = \sigma_{\theta\theta} + P \tag{9}$$

$$\tau_{xx} = \sigma_{xx} + P \tag{10}$$

$$\tau_{xr} = \sigma_{xr}. \tag{11}$$

Equation (1) can readily be integrated to give

$$v_r = -\frac{1}{r}\frac{\partial}{\partial x}\int_0^r rvdr. \tag{12}$$

By substituting eq (12) into eqs. (4) through (7), we get

$$\tau_{\theta\theta} = \frac{2\mu}{r^2}\frac{\partial}{\partial x}\int_0^r rvdr - 2\mu\frac{\partial v}{\partial x} \tag{13}$$

$$\tau_{\theta\theta} = -\frac{2\mu}{r^2}\frac{\partial}{\partial x}\int_0^r rvdr \tag{14}$$

$$\tau_{xx} = 2\mu\frac{\partial v}{\partial x} \tag{15}$$

$$\tau_{xr} = \tau_{rx} = \mu\frac{\partial v}{\partial r} - \frac{\mu}{r}\frac{\partial^2}{\partial x^2}\int_0^r rvdr. \tag{16}$$

When these expressions of stresses are substituted into the momentum eqs. (2) and (3) and P is eliminated, we obtain a single equation in v:

$$\left(\frac{\partial}{\partial r}\frac{1}{r}\frac{r}{\partial r}r - \frac{\partial^2}{\partial x^2}\right)\mu\frac{\partial v}{\partial r} + 2\left(\frac{\partial}{\partial r} + \frac{1}{r}\frac{\partial}{\partial r}r\right)\frac{\partial}{\partial x}\mu\frac{\partial v}{\partial x} + \left(-\frac{\partial}{\partial r}\frac{1}{r}\frac{\partial}{\partial r}\mu\frac{\partial}{\partial x}\right.$$
$$\left. - \frac{2}{r}\frac{\partial^2}{\partial r\partial x}\frac{\mu}{r} - \frac{2}{r^3}\frac{\partial}{\partial x}\mu + \frac{1}{r}\frac{\partial^2}{\partial x^2}\mu\frac{\partial}{\partial x}\right)\frac{\partial}{\partial x}\int_0^r rvdr = 0. \tag{17}$$

In this equation, operators written in the form of a product are to be computed from right to left.

Although eq. (17) looks very complex, a series solution can be obtained by giving apriori a viscosity profile

$$\mu = \mu_0 e^{\beta x}(1 + cr^2) \tag{18}$$

and by assuming a velocity profile in the form of

$$v = v_0 e^{\alpha x}(1 + a_1 r + a_2 r^2 + a_3 r^3 + \ldots) \tag{19}$$

where μ_0, β, c, v_0, and α are known constants and a_1, a_2, a_3, ... are unknown constants which should be determined to satisfy eq. (17) and the boundary condition at the thread surface discussed later.

If constants a_1, a_2, a_3, etc., can be determined in such a manner and if the series

$$1 + a_1r + a_2r^2 + a_3r^3 + \ldots$$

converges, then the solution eq. (19) may be considered to exist. The approximate condition under which the above series converges will be discussed later. Formulas (18) and (19) given a priori have the following interpretations. Figure 1 shows a cylindrical coordinate system taken with respect to a melt spinning thread, where x is the distance from the spinneret or, to be exact, the distance from the point where the thread diameter becomes largest due to die swell. In a previous study,[2] the author computed theoretically the thickness (cross-sectional area) $A(x)$ and temperature $t(x)$ of melt spinning thread as functions of x assuming that thread velocity and t are independent of r. These $A(x)$ and $t(x)$ curves can readily be converted into thread velocity $v(x)$ and shear viscosity $\mu(x)$ curves as shown in Figure 2 plotted on semilog graph paper. The two curves in Figure 2 can then be approximated by the tangents to the two curves in the neighborhood of $x = x$. $v_0e^{\alpha x}$ and $\mu_0e^{\beta x}$ in eqs. (18) and (19) are the expressions for the above two tangents to the $v(x)$ and $\mu(x)$ curves. The factor $(1 + a_1r + a_2r^2 + a_3r^3 + \ldots)$ in eq. (19) signifies the velocity variation in r direction when an axial symmetry is assumed.

In another previous study, the author[3] computed temperature profiles $t(x, t, \theta)$ within the thread assuming that macroscopic thickness profile $A(x)$ can be given independently. Computed temperature profiles showed that, in a typical industrial melt spinning, the temperature differential across the thread may reach as much as about 10% of the difference between the temperatures of spinneret and cooling air, and that the temperature profile across the thread is approximately parabolic in shape. This implies that an approximately parabolic distribution of shear viscosity exists across the thread. The factor $(1 + cr^2)$ in eq. (18) represents the viscosity profile with the c value derived from the temperature profile.

Strictly speaking, $v_0e^{\alpha x}$ and $\mu_0e^{\beta x}$ are velocity and shear viscosity at the thread core, but they can be substituted by the average values over the

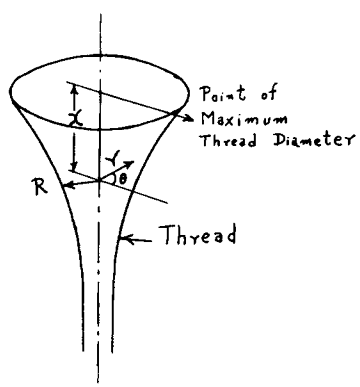

Fig. 1. Thread and cylindrical coordinates.

cross section, i.e., the macroscopic $v(x)$ and $\mu(x)$ values above, without causing excessive errors. Values of μ_0, β, v_0, α, and c are functions of x, but as far as the solution of eq. (17) is concerned, they can safely be assumed constants since it takes a change in x several orders of magnitude larger than thread radius R before these values change any significantly.

Equation (19) is considered to give the velocity field within a melt spinning thread under a steady state and at any distance from the spinneret, provided the above five parameters are given as functions of x. Equation (19), however, cannot cover the die swell region where the effect of the orifice is present.

Substituting eqs. (18) and (19) into eq. (17) and rearranging into a polynomial in r, we get

$$\begin{aligned} -a_1\frac{1}{r^2} &+ [9a_3 + (3c - \beta^2)a_1] + \Big[32a_4 + (16c + 4\alpha^2 \\ &+ 4\alpha\beta - 2\beta^2)\, a_2 + (2\alpha^2 + 6\alpha\beta)c + \frac{\alpha^2(\alpha+\beta)^2}{2}\Big] r \\ &+ \Big[75a_5 + (8\alpha^2 + 6\alpha\beta - 3\beta^2 + 45c)\, a_3 + \Big\{-(\alpha+\beta)^2 \\ &+ \frac{32}{3}\alpha(\alpha+\beta)c - \frac{5}{3}\alpha^2 c + \frac{\alpha^2}{3}(\alpha+\beta)^2\Big\} a_1\Big] r^2 \\ &+ \Big[144a_6 + 4(2\alpha^2 + 2\alpha\beta - \beta^2 + 24c)a_4 \\ &+ \Big\{(7\alpha^2 + 11\alpha\beta - 2\beta^2)c + \frac{1}{4}\alpha^2(\alpha+\beta)^2\Big\} a_2 \\ &+ \frac{1}{4}\alpha^2(\alpha+\beta)^2 c\Big] r^3 + \ldots = 0 \end{aligned} \quad (20)$$

For this relation to hold under any value of r, all coefficients on r^m have to be identically zero yielding the relations

$$a_1 = a_3 = a_5 = a_r = \ldots = 0 \quad (21)$$

$$a_4 = -\left(\frac{c}{2} + \frac{2\alpha^2 + 2\alpha\beta - \beta^2}{16}\right) a_2 - \frac{\alpha^2 + 3\alpha\beta}{16} c - \frac{\alpha^2(\alpha+\beta)^2}{64} \quad (22)$$

$$\begin{aligned} a_6 = &-\left(\frac{2}{3}c + \frac{2\alpha^2 + 2\alpha\beta - \beta^2}{36}\right) a_4 \\ &- \frac{1}{576}[4(7\alpha^2 + 11\alpha\beta - 2\beta^2)c + \alpha^2(\alpha+\beta)^2]a_2 - \frac{\alpha^2(\alpha+\beta)^2}{576} c. \end{aligned} \quad (23)$$

The value of a_2 which is not specified in the above relations is determined to satisfy the boundary condition at the thread surface.

We now proceed to state the boundary condition. Figure 3 shows a wedge-shaped segment of the thread whose slant face is the thread surface. If the static pressure P in eqs. (8) through (10) are expressed in gauge

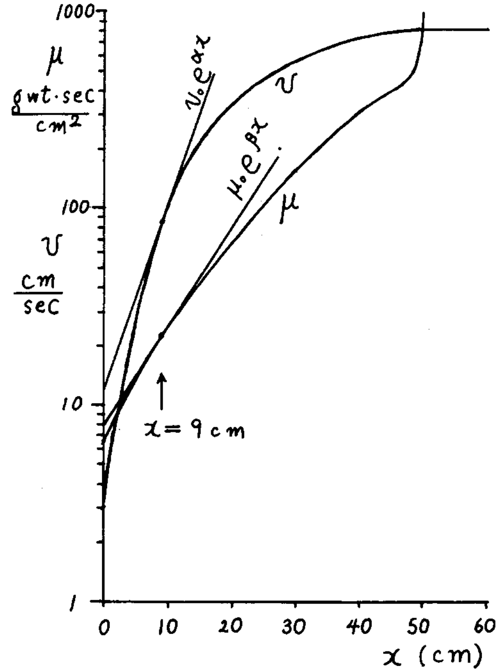

Fig. 2. Velocity v and shear viscosity μ as functions of distance x (Spinning conditions I)

pressure and if air drag force and surface tension are negligible, no external force is to act on the thread surface. If ξ is the gradient of yarn surface in the x-direction

$$\xi = \frac{dR}{dx} \tag{24}$$

then the force balance on the wedge in the x-direction is

$$r\Delta\theta \cdot \Delta x \cdot \sigma_{rx} = r \cdot \Delta\theta \cdot \xi \cdot \Delta x \cdot \sigma_{xx} \tag{25}$$

which reduces to

$$\sigma_{rx} = \xi\sigma_{xx}. \tag{26}$$

Similarly, the force balance in the r-direction yields

$$\sigma_{rr} + \frac{\Delta x}{2r}\,\xi\sigma_{\theta\theta} = \xi\sigma_{rx}. \tag{27}$$

The second term on the left-hand side vanishes as Δx approaches zero yielding

$$\sigma_{rr} = \xi\sigma_{rx}. \tag{28}$$

When eqs. (26) and (28) are converted into the expressions in τ using eqs. (8) through (11), we get

$$\tau_{rx} = \xi(\tau_{xx} - P) \tag{29}$$

$$\tau_{rr} - P = \xi\tau_{rx}. \tag{30}$$

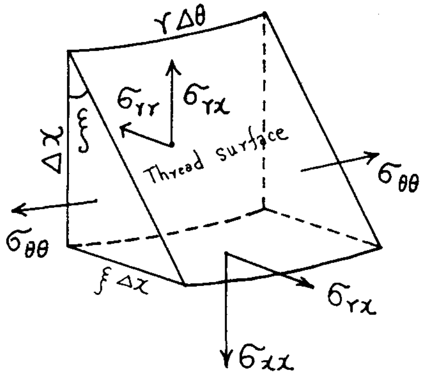

Fig. 3. Force balance on the thread surface.

When P is eliminated from these two relations, we obtain the statement of boundary condition

$$\xi(\tau_{xx} - \tau_{rr}) = (1 - \xi^2)\tau_{rx}. \tag{31}$$

ξ can also be expressed in terms of R and v by considering the equation of streamline within the thread:

$$\int_0^r rvdr = f = \text{const.} \tag{32}$$

By taking the total differential of this relation, we get

$$\frac{dr}{dx} = -\left(\frac{\partial f}{\partial x}\right)\bigg/\left(\frac{\partial f}{\partial r}\right) = -\left(\frac{\partial}{\partial x}\int_0^r rvdr\right)\bigg/(rv). \tag{33}$$

At the thread surface, this becomes

$$\xi = \frac{dR}{dx} = -\left(\frac{\partial}{\partial x}\int_0^r rvdr\right)_{r=R}\bigg/(rv)_{r=R}. \tag{34}$$

Substituting eqs. (19) and (21) into (34), we obtain

$$\xi = -\frac{\alpha R}{2}\,\frac{1 + \frac{1}{2}\,a_2R^2 + \frac{1}{3}\,a_4R^4 + \frac{1}{4}\,a_6R_6 + }{1 + a_2R^2 + a_4R^4 + a_6R_6 + }. \tag{35}$$

The value of a_2 can now be determined by considering eqs. (19), (21), (22), (23), (31), (35), (13), (15), and (16) simultaneously in unknown constants a_1, a_4, a_6, etc. Terms containing R^4 and higher become negligible in practice, and the solution for a_2 becomes

$$a_2 = \frac{-\alpha^2 + \left[\frac{1}{4}\,(\alpha^2 + 3\alpha\beta)c + \frac{1}{16}\,\alpha^2(\alpha + \beta)^2 - \frac{\alpha^4}{8}\right]R^2}{2 - \left[2c + \frac{-2\alpha^2 + 2\alpha\beta - \beta^2}{4}\right]R^2} \tag{36}$$

Other coefficients, a_4, a_6, etc., can be obtained by consecutively using eqs. (22), (23), etc.

Discussed next is the convergence of the series solution (19). When the equality in eq. (37) below is satisfied, a_2 becomes infinitely large:

$$c \leqq \frac{1}{R^2} - \frac{-2\alpha^2 + 2\alpha\beta - \beta^2}{8} \tag{37}$$

Therefore, the inequality above must hold. Since the second term is very small compared to $1/R^2$ under most conditions of melt spinning eq. (37) reduces to

$$R^2c < 1. \tag{38}$$

This is the first condition for the convergence of (19).

Under conceivable conditions of melt spinning, α and β do not exceed about 4 $\sec^{-1}$, and the thread radius R is at the most 0.05 cm, whereas the value of c may reach several thousand cm^{-2}. As a result, in eqs. (22), (23), etc., just the first terms on the right-hand side become significant as far as the convergence of eq. (19) is concerned:

$$a_4 = -\frac{c}{2}a_2, \qquad a_6 = -\frac{2}{3}\,ca_4, \ldots \qquad a_{j+1} = -ca_j. \tag{39}$$

The coefficients $-c/2$, $-(2/3)\ c$, ... are known to converge to $-c$. It follows that the series solution (19) converges approximately when the series

$$1 - cR^2 + c^2R^4 - c^3R^6 + c^4R^8 + \ldots. \tag{40}$$

converges. Series (40) converges when $cR^2 < 1$, which is identical to eq. (38). Referring to eq. (18), the condition $cR^2 < 1$ is equivalent to saying that viscosity at the thread surface is less than twice the viscosity at the thread core.

NUMERICAL EXAMPLES

Conditions in typical melt spinning of PP fibers are shown in Table I.

Spinning Conditions I

The macroscopic velocity $v(x)$ and shear viscosity $\mu(x)$ profiles computed for spinning conditions I are shown in Figure 2. Values of α, β, and R for the $x = 9$ cm point were derived from these curves. The c-value was derived from the temperature profile for the same point computed in a previous study[3]:

$$\begin{aligned} \alpha &= 0.242\ \text{cm}^{-1} \\ \beta &= 0.125\ \text{cm}^{-1} \\ R &= 5.6 \times 10^{-3}\ \text{cm} \\ c &= 4200\ \text{cm}^{-2} \end{aligned} \tag{41}$$

Parameter values (41) above satisfy the condition of convergence (38):

$$cR^2 = 0.132 < 1 \tag{42}$$

Values of a_2, a_4, etc., given by eq. (36), etc., for the above parameter values are listed in Table II. Also listed in Table II are the values for the case of $c = 0$ to know the effect of viscosity profile across the thread.

With or without the viscosity profile across the thread, the $a_2R^2 + a_4R^4$ values are so small in comparison to unity, the velocity profile under spinning conditions I is practically flat.

Spinning conditions II, shown in Table III, are for a water-quenched melt spinning of PET fiber in which the thinning of the thread completes in a narrow air gap of 2 cm. The quick thinning of the thread and the choice of $x = 0$ (vicinity of spinneret) make the values of α and R unusually large, tending in turn to make a_2 far larger than under most other spinning conditions conceivable. It should be noted here that the choice $x = 0$ does not make the solution cover the die swell region. It simply implies a neighborhood of spinneret where the effect of orifice has vanished.

Spinning Conditions II

The α, β, and R values were computed in the same manner as before from the $\mu(x)$ and $v(x)$ values computed in another study[7]:

$$\begin{aligned} \alpha &= 3.36 \text{ cm}^{-1} \\ \beta &= 0.117 \text{ cm}^{-1} \\ R &= 1.57 \times 10^{-2} \text{ cm} \end{aligned} \tag{43}$$

TABLE I
Spinning Conditions I

Density of polymer	0.83 g/cm³
Specific heat of polymer	0.70 cal/(g deg)
Activation energy of polymer	3500 deg
Spinneret temperature	270°C
Individual thread denier	8
Spinneret orifice diameter	0.6 mm
Horizontal cooling air speed	50 cm/sec
Take-up speed	500 m/min
Air temperature	30°C
Distance from spinneret	9 cm

TABLE II
Solution of Navier-Stokes Equations for Spinning Conditions I

	$c = 0$	$c = 4200$
a_2	-2.93×10^{-2}	-3.37×10^{-2}
a_4	1.73×10^{-4}	31.58
a_2R^2	-9.18×10^{-7}	-1.06×10^{-6}
a_4R^4	1.70×10^{-13}	3.10×10^{-8}
$a_2R^2 + a_4R^4$	-9.18×10^{-7}	-1.03×10^{-6}

Temperature differential across the thread is small since the vicinity of spinneret is now considered. However, an arbitrary c value of

$$c = 203 \qquad (cR^2 = 0.05) \tag{44}$$

is assumed just to add to the possible velocity gradient.

The solution is given in Table IV. Although the velocity differential across the thread is one order of magnitude bigger than in Table II, it is still negligibly small.

Proximity to the Pure Extensional Flow

As discussed in the previous section, a flat velocity profile can be assumed under most conditions of melt spinning. Therefore,

$$\frac{\partial v}{\partial r} = 0. \tag{45}$$

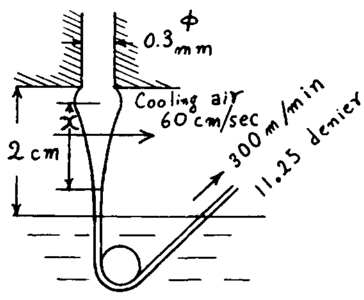

Fig. 4. Water-quenched melt spinning of PET.

TABLE III
Spinning Conditions II (See Fig. 4)

Density of polymer	1.33 g/cm^3
Specific heat of polymer	0.40 cal/(g deg)
Activation energy of polymer	3240 deg
Spinneret temperature	284°C
Individual thread denier	11.25
Spinneret orifice diameter	0.3 mm
Air gap	2 cm
Horizontal cooling air speed	60 cm/sec
Take-up speed	300 m/min
Air temperature	30°C
Water bath temperature	30°C
Distance from spinneret	0 cm

TABLE IV
Solution of Navier-Stokes Equations for Spinning Conditions II

a_2	-5.87
a_4	444
a_2R^2	-1.45×10^{-3}
a_4R^4	2.69×10^{-5}
$a_2R^2 + a_4R^4$	-1.43×10^{-3}

Under a flat velocity profile, eqs. (13) through (16) become

$$\tau_{rr} = -\mu \frac{\partial v}{\partial x} \tag{46}$$

$$\tau_{\theta\theta} = -\mu \frac{\partial v}{\partial x} \tag{47}$$

$$\tau_{xx} = 2\mu \frac{\partial v}{\partial x} \tag{48}$$

$$\tau_{rx} = -\frac{\mu\gamma}{2} \frac{\partial^2 v}{\partial x^2} \tag{49}$$

If, further, $\partial v/\partial x$ is a constant, the above expressions of stress becomes

$$\tau_{xx} = -2\tau_{rr} = -2\tau_{\theta\theta} = \text{const.} \tag{50}$$

$$\tau_{rx} = 0. \tag{51}$$

This is the case of the pure extensional flow. The ratio of τ_{rx} over τ_{xx} may be considered a measure of proximity to the pure extensional flow. Using eqs. (15), (16), (19), and (21), this ratio at the thread surface is expressed as

$$\tau_{rx}/\tau_{xx} = \frac{R\left[\left(2a_2 - \frac{\alpha^2}{2}\right) + \left\{4a_4 + \left(2c - \frac{\alpha^2}{4}\right)a_2 - \frac{\alpha^2 c}{2}\right\}R^2\right]}{2\alpha\{1 + (a_2 + c)R^2 + (a_4 + a_2 c)R^4\}} \tag{52}$$

$$= -5.85 \times 10^{-5} \text{ (Spinning Conditions I)}$$

$$= -2.48 \times 10^{-2} \text{ (Spinning Conditions II)}$$

Even under the highly unfavorable Spinning Conditions II, τ_{rx} is only 2.5% of τ_{xx}. Under the more common spinning conditions I, the ratio is a completely negligible $-0.57 \times 10^{-2}\%$. Under most spinning conditions, therefore, the velocity field within a thread can safely be assumed as pure extensional flow.

By using eqs. (19), (35), (48), and (49), it can be shown that in a pure extensional flow, the ratio τ_{rx}/τ_{xx} at the thread surface is

$$\tau_{rx}/\tau_{xx} = -\frac{\xi}{2}. \tag{53}$$

Also in a pure extensional flow, eq. (30) becomes

$$P = \tau_{rr}. \tag{54}$$

By eliminating P, τ_{xx} and τ_{rr} from eqs. (10), (46), and (48), we get

$$\sigma_{xx} = 3\mu \frac{\partial v}{\partial x}. \tag{55}$$

This is a statement of the well-known relation that the extensional viscosity is three times the shear viscosity.

CONCLUSIONS

1. The velocity field within a molten thread drawn and cooled in a melt spinning was analyzed by means of the equations of continuity and momentum for Newtonian liquids. By making several simplifying assumptions which the author[1] found to be valid in a previous study, a series solution in velocity v,

$$v = v_0 e^{\alpha x}(1 + a_2 r^2 + a_4 r^4 + a_6 r^6 + \ldots) \tag{56}$$

was obtained subject to the condition that shear viscosity μ is given as a function of distance x and radius r in the form of

$$\mu = \mu_0 e^{\beta x}(1 + cr^2). \tag{57}$$

2. Solution (56) above is the steady-state velocity field in the neighborhood of $x = x$. Values of v_0, α, μ_0, and β are obtainable by a separate "macroscopic" analysis in which μ and v are assumed independent of r. The c value is derived from the temperature profile across the thread computed separately using a technique developed by the author previously.[3] Although these four parameters are functions of x, they can safely be assumed to be constants in the present analysis.

3. Solution (56) converges when the condition

$$cR^2 < 1 \tag{58}$$

is satisfied.

4. The solution showed that even under unusually stringent spinning conditions, a_2, a_4, etc., are so small that the velocity profile across the thread in any melt spinning can safely be assumed to be flat.

5. The ratio τ_{rx}/τ_{xx} of stresses was found to be approximately equal to half of the gradient of thread surface in the x-direction and is 2.5% at the most. Therefore, the velocity field can further be assumed to be a pure extensional flow in most melt spinning.

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